

(UG1-861)

Roll No.

S.C.No.—2214103

B.C.A. EXAMINATION, 2023

(First Semester)

(Main/Re-appear/Improvement)

MATHEMATICS

22BCA103

Time : 3 Hours

Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) If :

 $A = \{a, b, c, d, e\}$

and $B = \{d, e, f, g\};$

find $(A - B) \cap (B - A).$

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Unit I

- (b) If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, find the values of a and b . 2
- (c) Write the minors and co-factors of the matrix $A = \begin{bmatrix} 5 & 20 \\ 0 & -1 \end{bmatrix}$. 2
- (d) If $f(x) = 2x + 1$, find the range if domain is $\{-1, 2, 3\}$ and hence find the function. 2
- (e) Evaluate : $\lim_{x \rightarrow 0} \frac{\sin ax}{bx}$. 2
- (f) Differentiate $(4x^3 - 5x^2 + 1)^4$ w.r.t. x . 2
- (g) Evaluate : $\int \frac{1}{x(1 + \log x)^2} dx$. 2
- (h) If $\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$, find the value of x . 2

2. (a) Prove that, for any three sets A , B and C , $(A \cap B) \cap C = A \cap (B \cap C)$. 8
- (b) For a certain test a candidate could offer Sanskrit or Urdu or both the subjects. Total number of students was 500, of whom 350 appeared in Sanskrit and 90 in both subjects. Use set operation to show : 8

- (i) How many appeared in Sanskrit only ?
- (ii) How many appeared in Urdu ?
- (iii) How many appeared in Urdu only ?

3. (a) Prove that :

$$\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z).8$$

- (b) Solve the following system of equations, using Cramer's rule :

$$x + y = 1$$

$$x + z = -6$$

$$x - y - 2z = 3.$$

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Unit II

4. (a) Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.

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- (b) Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \frac{5x+3}{7}$, $x \in \mathbb{R}$ is bijective function and hence find the inverse of f .

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5. (a) (i) Evaluate :

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x}$$

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- (ii) Evaluate :

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x}$$

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- (b) If a function ' f ' is defined as :

$$f(x) = \begin{cases} |x-4|, & x \neq 4 \\ 0, & x = 4 \end{cases}$$

Show that ' f ' is continuous everywhere except at $x = 4$.

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Unit III

6. (a) If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$, show that

$$\frac{dy}{dx} - y + \frac{x^n}{n!} = 0.$$

8

- (b) If $y = (x + \sqrt{x^2 + a^2})^n$, then find $\frac{dy}{dx}$.

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7. (a) Find $\frac{dy}{dx}$, if $y = \tan^{-1} \left(\frac{\sqrt{1+x^2} + 1}{x} \right)$.

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(b) If $(\cos x)^y = (\sin y)^x$, prove that :

$$\frac{dy}{dx} = \frac{\log \sin y + y \tan x}{\log \cos x - x \cot y} \quad 8$$

Unit IV

8. (a) Evaluate :

$$\int \tan^{-1}(\sec x + \tan x) dx. \quad 8$$

 (b) Evaluate :

$$\int x^2 \sin x dx. \quad 8$$

9. (a) Evaluate :

$$\int \frac{3x+1}{(x-1)^2(x+3)} dx. \quad 8$$

(b) Evaluate :

$$\int_0^{\pi/2} \frac{\sin \theta}{\sqrt{1+\cos \theta}} d\theta. \quad 8$$