S.C.No.—2009204

B.Sc. (Hons.) EXAMINATION, 2024

(Main) (Second Semester)

MATHEMATICS

BHM124

Discrete Mathematics-II

Time : 3 Hours *Maximum Marks : 60*

Note : Attempt *Five* questions in all. Q. No. **9** is compulsory. All questions carry equal marks.

1. (a) State and prove any *four* properties of lattice. **6**

(b) Show that the posets given below are lattices obtain the Hasse diagram of $(S_6, 1)$, $(S_8, 1)$, $(S_{24}, 1)$. **6**

2. (a) Describe lattice as an algebraic system. Show that a lattice with three or fewer elements is a chain. 6

(b) Dual of a complete lattice is complete. Explain. 6

3. (a) Define a Boolean Algebra and give one example of it. Prove uniqueness of complement in Boolean Algebra. 6

(b) In a Boolean Algebra, prove that : 6

$$(a \vee b) \vee c = a \vee (b \vee c).$$

4. (a) Find the conjuctive normal form for the Boolean function : 6

$$f = xyz + x'yz + xy'z' + x'yz'.$$

(b) Design a three-input minimal AND-OR circuit with the following truth table : 6

T = [A, B; C, L] = [00001111; 00110011; 01010101; 11001100].

5. (a) Show that there is no graph with 12 vertices and 28 edges in which the degree of each vertex is either 3 or 6. **6**

(b) If a graph has an Euler circuit, then prove that every vertex of the graph has even degree. **6**

6. (a) State and solve Konigsberg seven bridge problem. **6**

(b) Draw the graphs of the following adjacency matrices : **6**

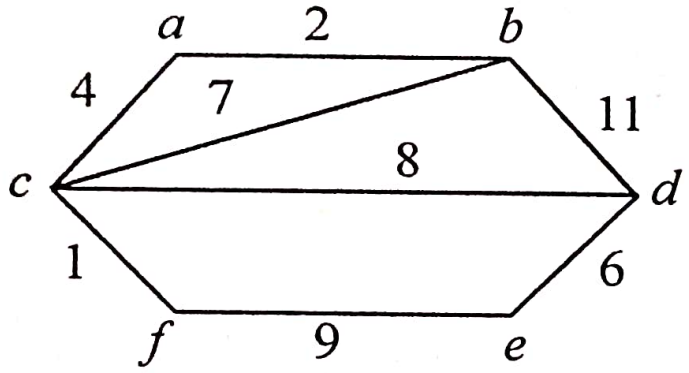
(i) $$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

(ii) $$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 0 & 2 & 1 \\ 1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix}.$$

7. (a) Does there exist a full binary tree with 12 internal vertices and 15 leaves. Explain. 6

(b) Show that the number of vertices is one more than the number of edges in a tree. 6

8. (a) Find a minimal spanning tree for the graph shown in figure given below : 6



(b) Let G be a connected graph with n vertices. Then G is a tree iff every edge of G is a bridge (cut edge). 6

9. (a) Define M-ary tree. 2

(b) Find the adjacency matrices of $K_{2,3}$. 2

(c) If a and b are in Boolean algebra, then $a \vee b = b \Leftrightarrow a' \vee b = 1$. **2**

(d) If G be a connected planar graph with v vertices, e eges where $v \geq 3$, then $e \leq 3v - 6$. **2**

(e) What is Euler circuit ? **2**

(f) Draw binary trees to represent :

(i) $2 - (3 \times x) + ((x - 3) - (2 + x)$

(ii) $ab - (c/(d + e))$. **2**