(b) If $\dim_F V = n$ and $T \in A(V)$ has index of nilpotency also n, then prove that a basis of V can be found in which matrix of T, all of which entries are zero except on the super diagonal, where they are all 1's. **8**

9. (a) If S, $T \in A(V)$ are nilpotent and ST = TS, then show that ST and $S \pm T$ are also nilpotent. **8**

(b) Let $T \in A(V)$ has all its distinct characteristics roots in F. Prove that a basis of B can be found in which the matrix of T is of the Jordan Canonical form. **8**

S.C.No.—M/22/18703101

M.Sc. EXAMINATION, 2022

(Batch 2018) (First Semester)

MATHEMATICS

18MTH101

Abstract Algebra-I

Time : 3 Hours *Maximum Marks :* 80

Note : Attempt *Five* questions in all. All questions carry equal marks.

1. (a) Prove that a commutative group having a composition series is finite.

(b) Give example of a solvable group which is not nilpotent.

(c) Find Sylow 2-subgroups of S_3 – the symmetric group of degree 3.

(d) Prove that a simple solvable group has prime order.

(e) Define Invariant Subspace.

(f) Define invariants of a linear transformation. Are they unique ?

(g) Define Radical of an ideal.

(h) Define UFD. **8×2=16**

Unit I

2. (a) Prove that any finite p-group is solvable. **8**

(b) Show that S_n is not solvable for $n > 4$. **8**

3. (a) State and prove Jordan-Holder Theorem. **8**

(b) Prove that a finite group is solvable if and only if its composition factors are cyclic groups of prime order. **8**

Unit II

4. (a) Prove that subgroup and homomorphic image of a nilpotent group is nilpotent. **8**

(b) State and prove Sylow's First Theorem. **8**

5. (a) Describe groups of order 15. **8**

(b) Let P be a Sylow p-subgroup of G and $x \in N(P)$ such that $O(x) = p^i$, for some $i \geq 0$. Then, $x \in P$. **8**

Unit III

6. (a) Let S and T be two ideals of a ring R, then prove that $S+T/S \cong T/S \cap T$. **8**

(b) Define Euclidean ring and prove that every Euclidean ring is a principal ideal domain. **8**

7. (a) Every finite non-zero integral domain is a field. **8**

(b) Prove that an element in a UFD is prime iff it is irreducible. **8**

Unit IV

8. (a) Let $\dim_F(V) = n$, and $T \in A(V)$ has all its characteristic roots in F. Prove that there is a basis of V in which matrix of T is triangular. **8**