Unit IV

8. (a) If V is a finite dimensional vector space over F, then T $\in$ A(V) is invertible iff constant term of minimal polynomial of T is non-zero.

(b) If T $\in$ A(V) and $p(x)$ be the minimal polynomial of T over F and $p(x)$ has all its roots in F, then prove that every root of $p(x)$ is a characteristic roof of T.

9. (a) If T $\in$ A(V) has all its characteristic roots in F, then there exists a basis of V in which matrix of T is triangular.

(b) If S and T are nilpotent transformations such that ST = TS, then ST and S + T are nilpotent transformations.

S.C.No.—M/22/21703101

M. Sc. EXAMINATION, 2022

(First Semester)

(Batch 2021)

MATHEMATICS

21MTH-101

Abstract Algebra-I

Time : 3 *Hours* *Maximum Marks* : 80

Note : Attempt *Five* questions in all. All questions carry equal marks.

1. (i) Define Composition Series.

(ii) Define Composition Series for the groups S_4 and Q_4.

(iii) State Zassenhaus Lemma.

(iv) Define Sylow-p-subgroup.

(v) Define PID and ED.

(vi) Define similarity of Matrices.

(vii) Describe all the groups of order p.

(viii) Define upper and lower central series.

Unit I

2. (a) Prove that every finite group having at least two elements has a composition series.

(b) State and prove Jordan Holder Theorem.

3. (a) Prove that every subgroup of a solvable group is solvable.

(b) Prove that every finite p group is solvable.

Unit II

4. (a) Prove that a normal series :

$$G = G_0 \supseteq G_1 \supseteq \ldots\ldots\ldots\ldots \supseteq G_r = \{e\}$$

of a group G is a central series of G iff $[G_{i-1}, G] \leq G_i$ for $1 \leq i \leq r$.

(b) If G is a group and $H \neq \{e\}$ be a normal subgroup of G contained in Z(G) such that G/H is nilpotent, then prove that G is nilpotent.

5. (a) State and prove Sylow's first theorem.

(b) Prove that every group of order p^2 is abelian.

Unit III

6. (a) If I_1 and I_2 are two ideals of a ring R, then prove that $I_1 + I_2$ is an ideal of R generated by $I_1 \cup I_2$.

(b) Prove that an ideal I of a ring of integers Z is a maximal ideal if and only if I is generated by some prime integer.

7. (a) If R is a commutative ring and I be an ideal of R, then prove that R/I is an integral domain iff I is a prime ideal.

(b) Prove that ring of polynomials of a field is a Euclidean ring.