Roll No.

# S.C.No.-21703102

# M.Sc. EXAMINATION, 2023

(Main/Re-appear/Improvement)

(2021/2022)

(First Semester)

### **MATHEMATICS**

21MTH-102

Differential Equations and Calculus of Variation

Time: 3 Hours

Maximum Marks: 80

Note: Attempt Five questions in all, selecting one question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

- 1. (i) Define Lipschitz Condition.
  - (if) What is Lagrange's identity for second order differential equation?

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- (iii) Define adjoint system.
- (iv) Define fundamental set.
- (v) What do you mean by index of a critical point ?
- (vi) What is the significance of Floquet theory for periodic system?
- (vii) Define Brachistochrone Problem.
- (viii) Find the proper and central field of externals for the functional  $\int_{0}^{4} y'^{2} dx$ .

 $8 \times 2 = 16$ 

#### Unit I

2 (a) Discuss the existence and uniqueness of a solution of the initial value problem:

$$\frac{dy}{dx} = y^{\frac{4}{3}}, \ y(x_0) = y_0.$$

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- Find the third approximation of the (b) solution of the problem  $\frac{dy}{dx} = y - x$ , y(0) = 2 by Picard's method.
- Let u(x) be any non-trivial solution of u'' + q(x)u = 0, where q(x) > 0 for all x > 0. If  $\int_{0}^{\infty} q(x)dx = \infty$ , then u(x) has infinitely many zero's on the positive 8 x-axis.
  - Consider the string problem:

$$\frac{d^2x}{dt^2} = f(t), x(0) = x(1) = 0,$$

then construct the Green's function.

#### Unit II

Show that the equation

$$P(x)y'' + Q(x)y' + R(x)y = 0$$

P(x)y'' + Q(x)y' + R(x)y = 0is self-adjoint if and only if P'(x) = Q(x).

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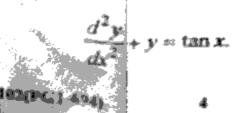
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(b) If φ(t) is the fundamental matrix of the homogeneous system x'(t) = A(t)x(t),  $t \in I$ , then  $\psi$  defined by :

$$\psi(t) = \phi(t) \int_{t_0}^{t} \phi^{-1}(\pm) H(\pm) dt, \quad t \in \mathbf{I} \text{ is a}$$

solution of the initial value problem of the non-homogeneous system  $\mathbf{t}'(t) = \Delta(t)\mathbf{t}(t) + \mathbf{B}(t), \ \mathbf{x}(t_0) = \mathbf{0}.$ 

- Define Wronskian. Evaluate Wronskian of the functions  $y_1(x) = \sin x$  and  $y_2(x) = \sin x - \cos x$  and hence conclude whether or not they are linearly independent.
  - Solve the following equation by the method of variation of parameters :



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### Unit III

- 6. (a) If (0, 0) is the only critical point of the linear system  $\frac{dx}{dt} = ax + by$ ,  $\frac{dy}{dt} = cx + dy$  and the roots of its characteristics equation are real and equal then prove that (0, 0) is a node.
  - (b) Find the nature and stability of the critical point of the system:

$$\frac{dx}{dt} = 2x + 4y, \frac{dy}{dt} = -2x + 6y.$$
Write a note on Liapunov's method to

- 7. (a) Write a note on Liapunov's method to determine stability.
  - (b) State and prove Bendixson non-existence theorem.

#### Unit IV

8. (a) Find the extremals of the following functional:

$$\int_{0}^{\pi} (y'^{2} - y^{2} + 4y\cos x)dx, \ y(0) = y(\pi) = 0.$$

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(b) Find the extremals of the isoperimetric problem;

$$J[y] = \int_{x_0}^{x_1} y'^2 dx$$

given that :

$$\int_{x_0}^{x_1} y dx = c, a \text{ constant}.$$

- (a) State and prove that fundamental lemma of calculus of variations.
  - (b) Find the xtremals of the functional:

$$J[y] = \int_{0}^{\pi/2} (y''^2 - y^2 + x^2) dx$$

subject to the boundary conditions: 8

$$y(0) = 1$$
,  $y'(0) = 0$ ,  $y(\frac{\pi}{2}) = 0$ ,  $y'(\frac{\pi}{2}) = 1$ .

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