(PG1-694) Roll No. ....

S.C.No.—21703102

M.Sc. EXAMINATION, 2023

(Main/Re-appear/Improvement)

(2021/2022)

(First Semester)

MATHEMATICS

21MTH-102

Differential Equations and Calculus of Variation

Time : 3 Hours *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

1. (i) Define Lipschitz Condition.

(ii) What is Lagrange's identity for second order differential equation ?

(iii) Define adjoint system.

(iv) Define fundamental set.

(v) What do you mean by index of a critical point ?

(vi) What is the significance of Floquet theory for periodic system ?

(vii) Define Brachistochrone Problem.

(viii) Find the proper and central field of externals for the functional $\int_0^4 y'^2 dx$.

8×2=16

Unit I

2. (a) Discuss the existence and uniqueness of a solution of the initial value problem : **8**

$$\frac{dy}{dx} = y^{\frac{4}{3}},\ y(x_0) = y_0.$$

(b) Find the third approximation of the solution of the problem $\frac{dy}{dx} = y - x$, $y(0) = 2$ by Picard's method. **8**

3. (a) Let $u(x)$ be any non-trivial solution of $u'' + q(x)u = 0$, where $q(x) > 0$ for all $x > 0$. If $\int_1^{\infty} q(x)dx = \infty$, then $u(x)$ has infinitely many zeros on the positive x-axis. **8**

(b) Consider the string problem :

$$\frac{d^2x}{dt^2} = f(t), \; x(0) = x(1) = 0,$$

then construct the Green's function. **8**

Unit II

4. (a) Show that the equation

$$P(x)y'' + Q(x)y' + R(x)y = 0$$

is self-adjoint if and only if $P'(x) = Q(x)$. **8**

(b) If $\phi(t)$ is the fundamental matrix of the homogeneous system $x'(t) = A(t)x(t)$, $t \in I$, then ψ defined by :

$$\psi(t) = \phi(t)\int_{t_0}^{t} \phi^{-1}(s)B(s)ds, \; t \in I$$ is a

solution of the initial value problem of the non-homogeneous system $x'(t) = A(t)x(t) + B(t)$, $x(t_0) = 0$. **8**

5. (a) Define Wronskian. Evaluate Wronskian of the functions $y_1(x) = \sin x$ and $y_2(x) = \sin x - \cos x$ and hence conclude whether or not they are linearly independent. **8**

(b) Solve the following equation by the method of variation of parameters : **8**

$$\frac{d^2y}{dx^2} + y = \tan x.$$

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Unit III

6. (a) If (0, 0) is the only critical point of the linear system $\frac{dx}{dt} = ax + by, \frac{dy}{dt} = cx + dy$ and the roots of its characteristics equation are real and equal then prove that (0, 0) is a node. **8**

(b) Find the nature and stability of the critical point of the system : **8**

$$\frac{dx}{dt} = 2x + 4y, \frac{dy}{dt} = -2x + 6y.$$

7. (a) Write a note on Liapunov's method to determine stability. **8**

(b) State and prove Bendixson non-existence theorem. **8**

Unit IV

8. (a) Find the extremals of the following functional : **8**

$$\int_0^{\pi} (y'^2 - y^2 + 4y\cos x)dx, \; y(0) = y(\pi) = 0.$$

(b) Find the extremals of the isoperimetric problem :

$$J[y] = \int_{x_0}^{x_1} y'^2 dx$$

8

given that :

$$\int_{x_0}^{x_1} ydx = c, \; a \text{ constant}.$$

9. (a) State and prove that fundamental lemma of calculus of variations. **8**

(b) Find the xtremals of the functional :

$$J[y] = \int_0^{\pi/2} (y''^2 - y^2 + x^2)dx$$

subject to the boundary conditions : **8**

$$y(0) = 1, \; y'(0) = 0, \; y\left(\frac{\pi}{2}\right) = 0, \; y'\left(\frac{\pi}{2}\right) = 1.$$