throw with one dice for a prize of Rs. 4,400 which is to be won by the player who first throws "6". If B has the first throw, what are their respective expectations ?

5. (a) Define Bionomial distribution. Obtain its mean and prove the following for moment :

$$\mu_{r+1} = pq\left[nr\mu_{r-1} + \frac{d\mu r}{dp}\right].$$

(b) A Poisson variate X is such that $P(X=1)=2$, $P(X=2)$, find $P(X=0)$, mean and variance. Also show that all cumulants are equal for Poisson distribution.

Unit III

6. (a) Prove that for a Normal distribution, mean deviation from mean is $\frac{4}{5}\sigma$ (approximately).

S.C.No.—M/22/21703105

M. Sc. EXAMINATION, 2022

(First Semester)

(Batch 2021)

MATHEMATICS

21MTH-105

Mathematical Statistics

Time : 3 *Hours* *Maximum Marks :* 80

Note : Attempt *Five* questions in all. All questions carry equal marks.

1. (a) Define Trial, Independent events and Equally likely events.

(b) Define a continuous random variable along with *one* example.

(c) Find the expectation of the number on a dice when thrown.

(d) Define geometric distribution. Obtain its M.G.F.

(e) Obtain mean deviation about mean for uniform distribution.

(f) State central limit theorem.

(g) Define Consistency and Efficiency.

(h) Explain types of errors. **8×2=16**

Unit I

2. (a) For n events A_1, A_2....., An; prove that

$$P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1).$$

(b) A and B throw alternately with a pair of dice. One who first throws a total of 9 wins. What are their respective chances of winning if A starts the game ?

(c) A problem in statistics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem is solved ? **6+5+5=16**

3. (a) State and prove Baye's theorem on probability.

(b) Define discrete random variable and sample space. A random variable X has the following probability function :

X:	−2	−1	0	1	2	3
$f(x)$:	k	0.3	$2k$	0.2	0.1	$3k$

Find the value of k, $P(X<0)$; $P(-2<X\leq 2)$. Also determine the distribution function of X.

Unit II

4. (a) The joint probability density function of two random variables X and Y is :

$$f(x,y) = \begin{cases} Kx(x-y), & 0<x<2; \quad -x<y<x \\ 0, & \text{elsewhere} \end{cases}$$

Find constant K and obtain the marginal distributions of X and Y, the conditional distribution of Y for $X=x$ given.

(b) Define mathematical expectation and moment generating function. A and B

(b) Let X have the p.d.f.

$$f(x)=\begin{cases}\theta e^{-\theta x}; & x>0\\ 0; & \text{otherwise}\end{cases}$$

Show that if the positive part of x-axis is divided into intervals of equal length h starting at the origin, then the probabilities that X will lie in successive intervals form a G.P. with common ratio $e^{-\theta h}$.

7. (a) State and prove Weak law of large numbers.

(b) Show that for a Uniform distribution; $f(x)=\frac{1}{2a}$; $-a<x<a$. The M.G.F. about origin is $\frac{\sin h\, at}{at}$. Also deduce the moments about mean for this distribution.

Unit IV

8. (a) Explain the following : Sufficiency, Null and Alternate Hypothesis, Critical region and level of significance.

(b) What do you mean by unbiased estimator ? Let X be distributed in Poisson form with parameter θ. Show that the unbiased estimator of $e^{-(K+1)\theta}$, $K > 0$ is $T(X) = (-K)^X$ so that $T(X) < 0$ if X is odd and $T(X) > 0$ if X is even.

9. (a) A survey of 320 families with 5 children each revealed the following distribution :

No. of boys :	5	4	3	2	1	0
No. of girls :	0	1	2	3	4	5
No. of families :	14	56	110	88	40	12

Is this result consistent with the hypothesis that male and female births are equally probable ? (Given $\psi^2_{0.05}$ for $5d \cdot f \cdot = 11.07$).

(b) The 9 items of a sample had the following values :

45, 47, 50, 52, 48, 47, 49, 53, 51

Does the mean of 9 items differ significantly from the assumed population mean 47.5 ? (Given t for 8 $d \cdot f \cdot$ at 5% level of significance is 2.31).