(PG86)   Roll No. ....

S.C.No.—21703201

M. Sc. EXAMINATION, 2023

(Second Semester)

(Main/Reappear/Improvement)

(2021/2022)

MATHEMATICS

21MTH-201

Abstract Algebra–II

Time : 3 Hours   *Maximum Marks :* 80

Note : Attempt any *Five* questions. All questions carry equal marks.

Unit I

1. (a) Define simple and cyclic module.

   (b) Prove that KerT is submodule of M, where T : M → N is a module homomorphism.

   (c) Give an example of a ring which is neither Noetherian and nor Artinion.

   (d) Prove that in an Artinion Commutative ring with unity, every ideal is maximal.

   (e) Find the splitting field of the polynomial $x^3 - 2$ over the field Q and it basis.

   (f) Show that Polynomial $x^4 + x^2 + 1$ is reducible or irreducible over Z.

   (g) Prove that G (K, F) is a subgroup of AutK.

   (h) Let [K : F]= 2 then K/F is a Normal extension. 8×2=16

Unit II

2. (a) State and prove Fundamental theorem of module homomorphism. 8

   (b) Prove that a R-module N is direct sum of its submodule $N_1, N_2, N_3, \ldots\ldots N_m$ iff

   (i) $N = N_1 + N_2 + N_3 + \ldots\ldots + N_m$

   (ii) $N_i \cap (N_1 + N_2 + N_3 + \ldots + N_{i-1} + N_{i+1} + \ldots\ldots + N_m) = \{0\}$. 8

3. (a) Let N be a finitely generated free module over a commutative ring R with unity then all basis of N are finite and having same number of elements. 8

(b) State and prove the fundamental theorem of finite generated modules over a principal ideal domain. 8

Unit III

4. (a) Let N be a R-module then N is Noetherian iff every non-empty family of R-submodules of N has a maximum element. 8

(b) Let $N_1 \times N_2 \times N_3 \times \ldots\ldots \times N_m$ be Artinion submodules of a module N then $\sum_{i=1}^{m} N_i$ is also Artinion. 8

5. Show that if R is a Noetherian ring then the polynomial ring R [x] is so and conversely. 16

Unit IV

6. (a) Prove that finite extension of finite extension is also a finite extension. 8

(b) Let K/F be any extension and $a \in K$ is algebraic over F. Let $p(x) \in F[x]$ be the minimal polynomial of a. Then :

$$F[x] / \langle p(x) \rangle \cong F(a) = F(a).$$ 8

7. (a) Prove that $\sin m^0$ is an algebraic integer for every integer m. 8

(b) Show that $\sqrt{3} + \sqrt[3]{5}$ is algebraic over Q of degree 6. Also find out its splitting field degree. 8

Unit V

8. (a) A field F is finite iff $F^* = F - \{0\}$ is a multiplicative cyclic group. 8

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(b) Let ch. $F = p > 0$. Prove that the element 'a' in some extension of F is separable iff $F(ap) = F(a)$. 8

9. State and prove Fundamental theorem of Galois Theory. 16