(PG70) Roll No.

S.C.No.—23703201

M.Sc. EXAMINATION, 2024

(For Second Semester)

(Batch 2023)

(Main)

MATHEMATICS

(23MTH-N-201)

Abstract Algebra-II

Time : 3 Hours *Maximum Marks : 80*

Note : Attempt *Five* questions in all, Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Define left Artinian ring.

(b) Define Noetherian module.

(c) Define separable polynomial.

(d) Define the characteristic of a ring.

(e) Define simple extension of a field.

(f) Define Splitting field of a polynomial.

(g) Define Algebraic extension of a field.

(h) Define Fixed field.

2. (a) Prove that every abelian group G is a module over the ring of integer Z.

(b) Let f be a homomorphism of R-module M into an R-module N. Show that f is an isomorphism iff Ker $f = \{0\}$.

3. (a) Let M be a finitely generated free module over a commutative ring R. Prove that all bases of M are finite.

(b) Let M_1, M_2 be free R-module then prove that $M_1 \times M_2$ is also a free module.

4. Let N be a R-submodule of a R-module M. Show that M is Noetherian iff N and M/N are Noetherian.

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5. Prove that in a Noetherian ring, sum of Nilpotent ideals is again a Nilpotent ideal.

6. (a) Find the splitting field and its degree for the polynomial $x^4 - 3$ over Q.

(b) Let $K|_F$ be any extension and $f(x) \in F[x]$, show that the element $a \in K$ is a root of polynomial $f(x)$ iff $(x - a) \mid f(x)$ in $K[x]$.

7. (a) Let $p(x) \in F[x]$ be the polynomial of degree $n \geq 1$, prove that there exists an extension E of F containing all the roots of $p(x)$ and $[E : F] \leq n!$.

(b) If $a \in K$ is algebraic over F of odd degree, show that $F(a) = F(a^2)$.

8. If E is a finite extension of a field F, prove that E is simple extension of F iff there are only a finite number of subfields of E containing F.

9. (a) Show that the group of automorphism of a field F with p^n elements is cyclic of order n and generated by φ, where $\varphi(x) = x^p$, $x \in F$.

(b) Let F be a finite field. Show that :

(i) The characteristic of F is a prime number p and F contains a subfield $F_p \cong Z/\langle p \rangle$.

(ii) The number of elements of F is p^n for some positive integer n.

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