(PG88)

Roll No. ....

S.C.No.—21703202

M. Sc. EXAMINATION, 2023

(Second Semester)

(2021/2022) (Main/Re-appear/Improvement)

MATHEMATICS

21MTH-202

Complex Analysis

Time : 3 Hours   Maximum Marks : 80

Note : Attempt any *Five* questions. All questions carry equal marks.

Unit I

1. (a) Define radius of convergence of a power series.

   (b) Write any *four* properties of an analytic function.

   (c) Write Riemann's definition of complex integration.

(d) State Cauchy's integral formula for higher order derivatives.

(e) Define zero of an analytic function.

(f) Write down various types of singularities of an analytic function.

(g) Write down formulas to calculate residues.

(h) Define rotation, translation and magnification.

Unit II

2. (a) Derive C-R equations in Polar co-ordinates.

   (b) If $f(z)$ is an analytic function with constant modulus, then prove $f(z)$ is constant.

3. Prove that a power series represents an analytic function inside its circle of convergence.

Unit III

4. (a) State and prove Cauchy's fundamental theorem.

   (b) Derive Cauchy's integral formula for simply connected region.

5. (a) Let $f(z)$ be analytic in region D and let $F(z) = \int_{z_0}^{z} f(w)dw$; where z_0 is any fixed point in D. Then prove that function $F(z)$ is analytic in D and its derivative is $f(z)$.

(b) State and prove Liouville's theorem.

Unit IV

6. (a) State and prove Schwarz Lemma.

(b) State and prove Cassorati-Weierstrass theorem.

7. (a) State and prove Minimum Modulus Principle.

(b) Evaluate the integral :

$$\int_{0}^{\Pi} \frac{ad\theta}{a^2 + \sin^2\theta}, a > 0.$$

Unit V

8. (a) Evaluate the integral :

$$\int_{-\infty}^{\infty} \frac{dx}{(a^2 + x^2)(b^2 + x^2)}, a > 0 \text{ and } b > 0.$$

(b) State and prove Argument principle.

9. (a) Prove that every Bilinear transformation is the resultant of Bilinear transformations with simple geometric imports.

(b) Find the image of $x^2 + y^2 - 4y + 2 = 0$ under the mapping :

$$w = \frac{z - i}{iz - 1}.$$