(PG92) Roll No. ....

S.C.No.—21703204

M.Sc. EXAMINATION, 2023

(Main/Re-appear/Improvement)

(2021/2022)

(Second Semester)

MATHEMATICS

21MTH-204

Measure and Integration Theory

Time : 3 Hours *Maximum Marks : 80*

Note : Attempt any *Five* questions. All questions carry equal marks.

Unit I

1. (a) Prove that outer measure of null set is zero.

(b) Prove that countable intersection of measurable set is measurable.

(c) Prove that set of type F_σ are measurable

(d) State Fatou's Lemma.

(e) State Monotone Convergence Theorem.

(f) State Vitali's Covering Lemma.

(g) State Fundamental theorem of Differential Calculus.

(h) State Jensen's inequality.

Unit II

2. (a) Prove that a finite union of measurable set is measurable.

(b) Let $\{E_i\}$ be an infinite increasing sequence of measurable sets then prove that :

$$m\left(\bigcup_{i+1}^{\infty} E_i\right) = \lim_{n\to\infty} E_n$$

3. State and prove Egoroff's theorem.

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Unit III

4. State and prove Lebesgue Dominated Convergence theorem.

5. (a) Let f be bounded Riemann integrable function on $[a, b]$, then prove that it is Lebesgue integrable and

$$R\int_a^b f(x)dx = \int_a^b f(x)dx.$$

(b) Evaluate the Lebesgue integral of the function $f:[0, 1] \to R$:

$$f(x) = \begin{cases} \dfrac{1}{x^{1/3}} & \text{if} \quad 0 < x \leq 1 \\ 0 & \text{if} \quad x = 0 \end{cases}$$

Unit IV

6. State and prove Jordan's decomposition theorem.

7. (a) Prove that if f is an absolutely continuous function on $[a, b]$, f has a derivative a.e.

(b) If f and g are absolutely continuous function on $[a, b]$, prove that :

$$\int_a^b f(t)g'(t)dt + \int_a^b f'(t)g(t)dt$$
$$f(b)g(b) - f(a)g(b).$$

Unit V

8. (a) Prove that a twice differentiable function φ is convex iff $\varphi'' \geq 0$ on (a, b).

(b) State and prove Hölder's inequality.

9. State and prove the Riesz-Fischer theorem.

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