Roll No. ....

(PG90)

S.C.No.—21703203

M.Sc. EXAMINATION, 2023

(Main/Re-appear/Improvement)

(2021/2022)

(Second Semester)

MATHEMATICS

21MTH-203

Partial Differential Equations

Time : 3 Hours *Maximum Marks : 80*

Note : Attempt any *Five* questions. All questions carry equal marks.

1. (a) Solve $z = e^{ax+by} f(ax - by)$ by eliminating the arbitrary functions.

(b) Find the complete integral of $q = xyp^2$.

(c) Discuss the wave motion along an infinite string with zero initial displacement.

(d) Find out the necessary condition for the Neumann problem.

(e) State Duhamel's Principle.

(f) State Kelvin's Inversion theorem.

(g) Classify the partial differential equation :

$$2\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0.$$

(h) Discuss the Monte Carlo Method in brief.

8×2=16

Unit I

2. (a) Find the solution of $(xz + y^2)p + (yz - 2x^2)q + 2xy + z^2 = 0$. **8**

(b) Find out the condition that $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ are to be compatible. **8**

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3. (a) The deflection of a vibrating string of length l, is goverened by the partial differential equation $\frac{\partial^2 y}{\partial x^2} = C^2 \frac{\partial^2 y}{\partial t^2}$. The initial velocity is zero and the initial displacement is given by

$$y(x,0) = \frac{x}{l},\ 0 < x < \frac{l}{2}.$$
$$\frac{l-x}{l},\ \frac{l}{2} < x < l$$

Find the deflection of string at any instant of time. 8

(b) Find complete integral of $z^2p^2y + 6pzxy + 2zqx^2 + 4x^2y = 0$. 8

Unit II

4. (a) State and prove Minimum principles. 8

(b) Solve the Dirchlet problem for a exterior circle. 8

5. (a) Find out the solution of The Dirchlet problem for the upper half plane. 8

(b) Prove that the solution of the Neumann problem is unique up to the addition of a constant. 8

Unit III

6. (a) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t},\ 0 < x < l, t > 0$ given that $u(0, t) = u(l, t = 0)$ and $u(x, 0) = x(l - x),\ 0 \le x \le l$. 8

(b) Show that the surfaces $x^2 + y^2 + z^2 = cx^{2/3}$ can form an equipotential family of surfaces and find the general form of potential function. 8

7. (a) Find out the Green's function for a unit ball. 8

(b) State and prove representation formula using Green's function. 8

Unit IV

8. (a) Reduce the equation $x^2r - y^2t = 0$ to canonical form. **8**

(b) Solve :

$u_{xx} = u_t - 3u$, $u(0, t) = 0$, $u(2, t) = 0$ for $t > 0$ and $u(x, 0) = 1$ for $0 \le x \le 2$, by the implicit method. **8**

9. (a) Obtain the solution :

$$u_{yy} + \cos y u_{xx} = 0, \quad 0 < x < \frac{\pi}{2},$$

$$0 < y < \frac{\pi}{2}, u(y, x) = f(y, x)$$

on the boundary of the square, by the Monte Carlo Method. **8**

(b) Discuss the Explicit Finite Difference Method by giving an example. **8**