(PG3-340) Roll No. ....

S.C.No.—21703302

M.Sc. EXAMINATION, 2023

(Main/Re-appear/Improvement)

(2021)

(Third Semester)

MATHEMATICS

21MTH-302

Differential Geometry

Time : 3 Hours *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Questions)

1. (a) Find the equation of tangent to the curve $\vec{r} = (A\cos\theta, A\sin\theta, B\theta)$; where A and B are constants, $-\infty < \theta < \infty$.

(b) Define torsion of a curve. Also evaluate $\vec{r}', \vec{r}'''$.

(c) Find envelope for the surface $x^2 + y^2 = 8c(z-c)$, where c is parameter.

(d) What do you mean by edge of regression of the envelope ?

(e) Write the condition that the two directions given by $Pdu^2 + Qdudv + Rdv^2 = 0$ are orthogonal.

(f) What do you mean by directions on a surface ?

(g) Define minimal surface and Gauss curvature.

(h) Prove that two geodesics at right angles have their torsions equal in magnitude but opposite in sign.

Unit I

2. (a) Find curvature and principal normal for the curve :

$$x = 4b\cos^3\theta, y = 4b\sin^3\theta, z = 3c\cos 2\theta,$$

where b and c are constants.

(b) Show that :

$$\left[\vec{b}',\vec{b}'',\vec{b}'''\right]=\tau^3(\kappa'\tau-\kappa\tau')$$

3. (a) Prove that in order that principal normal of a curve be binormal of another, the relation $a(\kappa^2+\tau^2)=\kappa$ must hold, where a is constant.

(b) If the tangent and binormal at a point of a curve make angles α and β, respectively with a fixed direction, show that :

$$\frac{\sin\beta d\beta}{\sin\alpha d\alpha}=-\frac{\tau}{\kappa}.$$

Unit II

4. (a) If C be any given curve and C_1 denotes the locus of spherical curvature, show that the product of curvatures of these two curves is equal to the product of their torsions.

(b) Prove that any tangent plane to the surface $a(x^2+y^2)+xyz=0$ meets it again in a conic whose projection on the plane of xy is a rectangular hyperbola.

5. (a) Find the edge of regression of the envelope of the family of planes. $x\sin\theta-y\cos\theta+z=a\theta$; θ being the parameter.

(b) The normal at a point P of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meets the co-ordinate planes in A_1, A_2, A_3. *Prove that the ratios* $PA_1 : PA_2 : PA_3$ are constant.

Unit III

6. (a) If the parametric curves are orthogonal, show that the differential equation and lines on the surface cutting the curves u = constant at a constant angle β is

$$\frac{du}{dv}=\tan\beta.\sqrt{\frac{G}{E}}.$$

(b) Show that on the surface given by $x = a(u + v)$, $y = b(u - v)$, $z = uv$, the parametric curves are straight lines. Also calculate the fundamental magnitudes of this surface.

7. (a) Prove that :

$$T^2 \bar{r}_1 = (FE - EN)\bar{n}_1 + (EM - FL)\bar{n}_2,$$

where symbols have their usual meanings.

(b) If L, M, N vanish at all points of the surface, then show that surface is a plane.

Unit IV

8. (a) Derive the equation for principal directions at any point on a surface.

(b) Prove that the surface of revolution given by :

$x = u \cos v, y = u \sin v,$

$$z = a \log\left\{u + \sqrt{u^2 - a^2}\right\}$$

is a minimal surface. Also obtain first and second curvatures.

9. (a) If κ and τ are the curvature and torsion of a geodesic, prove that :

$$\tau^2 = (\kappa - \kappa_a)(\kappa_b - \kappa)$$

(b) Find the first integral of the differential equation of geodesic on the surface $x = u\cos v, y = u\sin v, z = cv$.

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