S.C.No.—21703303

M. Sc. EXAMINATION, 2023

(Third Semester)

(Main/Re-appear/Improvement) (2021)

MATHEMATICS

21MTH-303

Mechanics of Solids—I

Time : 3 Hours *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (a) Define zero tensor and equality of two tensors.

(b) Evaluate :

$$\delta_{jk}\epsilon_{ijk} \ \& \ \epsilon_{ijk}\epsilon_{pjk}$$

(c) Explain plane stress.

(d) What do you mean by principal directions of stress ?

(e) Define infinitesimal affine transformation.

(f) Explain uniform dilatation.

(g) Are the principal axes of stress coincide with those of strain for an isotropic elastic medium ?

(h) Give statement of Clapeyron's theorem.

Unit I

2. (a) Define symmetric and skew-symmetric tensors. If A_{ij} is a skew-symmetric second order tensor, show that :

$$(\delta_{ij}\delta_{lk} + \delta_{il}\delta_{jk})A_{ik} = 0$$

(b) Using tensorial transformation, show that curl of a first order tensor is a vector.

(c) If a_{ij} and b_{mk} are second order tensors, show that $a_{ij}b_{ij}$ is a scalar and $a_{ij}b_{mj}$ is a second order tensor.

3. (a) Prove the following :

$$\epsilon_{ijm}\epsilon_{klm} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}$$

(b) Prove that eigen values of a symmetric second order tensor are real and eigen vectors are orthogonal.

Unit II

4. (a) State and derive Cauchy's equations of equilibrium.

   (b) Discuss Mohr's diagram for finding maximum shearing stress when two principal stresses are equal.

5. (a) Prove symmetry of stress tensor.

   (b) The state of stress at a certain point of a body is given by :

$$\begin{bmatrix} 2 & 4 & 3 \\ 4 & 0 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

   Find the stress vector acting on a plane passing through point and parallel to the plane :

$$2x + y + 2z = 4$$

Unit III

6. (a) Define strain tensor. Give geometrical interpretation of its shear component e_{13}.

   (b) Write a short note on strain quadric of Cauchy.

7. (a) Show that the necessary and sufficient condition for an infinitesimal transformation to represent a rigid body motion is that the coefficient α_{ij} are skew-symmetric

   (b) Refer the quadric of deformation to a set of principal axes and discuss the nature of deformation when the quadric is an ellipsoid and when it is hyperboloid. Also explain physical significance of cubical dilatation.

Unit IV

8. (a) State generalized Hooke's law. Find how many constants will be there, if there is one-plane of symmetry.

(b) Show that if $\sigma = 0$, then $\lambda = 0$, $\mu = E/2$, $K = E/3$ and Hooke's law is expressed by :

$$\tau_{ij} = \frac{E}{2}\left(u_{i,j} + u_{j,i}\right)$$

Also show that : **10+6**

$$\tau_{ij,ij} = \frac{1}{2}\left(\tau_{ii,jj} + \tau_{jj,ii}\right)$$

9. (a) Prove that :

$$\nabla^2\theta = -\frac{(1+\sigma)}{(1-\sigma)} \operatorname{div} \vec{F}$$

where symbols have their usual meanings.

(b) Explain physical interpretation of Bulk modulus (k) and modulus of rigidity (μ).

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