S.C.No.—21703301

M. Sc. EXAMINATION, 2023

(Third Semester)

(Main/Re-appear/Improvement) (2021)

MATHEMATICS

21MTH-301

Topology

Time : 3 Hours *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Let X = N, the set of natural numbers and $\mathfrak{J}$ consists of ϕ, X and all subsets of X of the form {1, 2,.....n}, $n \in$ N. Show that $\mathfrak{J}$ is a topology on X.

(b) Let (X, $\mathfrak{J}$) be a top space, then a subfamily β of $\mathfrak{J}$ is a base for $G \in \mathfrak{J}$ and any $x \in G$, there exist a $B \in \beta$ such that $x \in B \subseteq G$.

(c) If C is connected subset of a topological space (X, $\mathfrak{J}$), which has a separation X = A | B, then prove that either $C \subseteq A$ or $C \subseteq B$.

(d) Every closed subset of a compact space is compact.

(e) Prove that the property of a topological space being a T_0-space is a topological property.

(f) Define Component.

(g) Define locally compact space.

(h) Give example of a second axiom space.

8×2=16

Section I

2. (a) Let S be the family of sets, then the family β of finite intersection of members

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of S is a base for a topology on a set $X = \cup\{S : S \in S\}$ and prove that topology is the smallest topology containing S. 8

(b) Prove that in a topological space $(X, \mathfrak{J})$, a subset F is closed iff $X - F$ is open. 8

3. (a) Let $(X, \mathfrak{J})$ be a topological space, then for every $A \subseteq X$, the interior of A satisfies the following in $(X, \mathfrak{J})$: 8

(i) $\phi^\circ = \phi$, $X^\circ = X$

(ii) $(A^\circ)^\circ = A^\circ$

(iii) If $A \subseteq B$, then $A^\circ \subseteq B^\circ$

(iv) $(A \cap B)^\circ = A^\circ \cap B^\circ$

(v) $A^\circ \cup B^\circ \subseteq (A \cup B)^\circ$.

(b) Let $(X, \mathfrak{J})$ be a topological space and $A \subseteq X$, then $\overline{A} = A \cup d(A)$. 8

Section II

4. (a) Show that components are closed. Also show that any continuous image of connected space is connected. 8

(b) Let (X, T) and (X^*, T^*) be two topological spaces. Show that a one to one mapping f of X on to X^* is homeomorphism if $f(C(E)) = C(f(E))$, where C represents the closure of the sets in corresponding space, for very $E \subseteq X$. 8

5. (a) Show that a subset A of the real line R containing at least *two* points is connected iff it is an interval. 8

(b) Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X. 8

Section III

6. (a) Prove that a topological space $(X, \mathfrak{J})$ is compact iff any family of closed sets having finite intersection property has non-empty intersection. 8

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(b) Prove that every compact subset of a topological space is countably compact. 8

7. (a) Let $(X, \mathfrak{J})$ be a topological space and let $X^* = X \cup \{\infty\}$ where ∞ is any object not belonging to X. Let $\mathfrak{J}^*$ consists of all those subsets of X^* which are of the following types :

(i) U, where U is an open subset of X

(ii) $X^* - C$, where C is a closed compact subset of X.

Prove that $\mathfrak{J}^*$ is a topology X^* and $(X^*, \mathfrak{J}^*)$ is a compact topological space. 8

(b) Prove that one point compactification X^* of a topological space X is a Hausdorff space iff X is locally compact Hausdorff space. 8

Section IV

8. (a) Let $(X, \mathfrak{J})$ be a topological space which is T_1 as well as first countable and E be a subset of X. Let $x \in X$, then prove that x is a limit point of E iff there exists a sequence of distinct point of E converging to x. 8

(b) State and prove Baire Category theorem. 8

9. (a) Prove that a topological space $(X, \mathfrak{J})$ is a T_2-space iff for all $x \in X$, $\cap\{\overline{U}_x ; U_x \in \mu_x\} = \{x\}$, where μ_x is the neighborhood system of x in the given topology 8

(b) State and prove Tietze extension theorem. 8

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