M. Sc. EXAMINATION, 2023

(Fourth Semester)

(Main/Reappear/Improvement)

(2021)

MATHEMATICS

21MTH-404

Advanced Fluid Dynamics

Time : 3 Hours *Maximum Marks :* 80

Note : Attempt any *Five* questions. All questions carry equal marks.

Unit I

1. (a) Write down the statement of Milne-Thomson Circle theorem.

(b) Define Karman Vortex Street.

(c) Define Subsonic, Sonic and Supersonic flows.

(d) What is Shock Waves ?

(e) Define Plane Poiseuille flow.

(f) Write down the relation between the stress and gradients of velocity.

(g) Define Pressure Co-efficient.

(h) What is Prandtl's Boundary Layer ?

8×2=16

Unit II

2. (a) Find out the image system for a source in a circular cylinder of a radius a. 8

(b) State and prove the Blasius theorem. 8

(a) With a regard boundary in the form of the circle $(x+a)^2+(y-4a)^2=8a^2$, there is a liquid motion due to a doublet of strength μ at the point $(0, 3a)$ with its

axis along the axis of y. Show that the velocity potential is :

$$\mu\left\{\frac{4(x-3a)}{(x-3a)^2+y^2}+\frac{(y-3a)}{(y-3a)^2+x^2}\right\}. \quad 8$$

(b) To show that the motion due to a set of line vortices of strength k at points $z=\pm na$, $(n=0, 1, 2, 3,...)$ is given by the relation $w=\left(\frac{ik}{2\pi}\right)\log\sin\frac{\pi z}{a}$. Also find out the velocity components and streamlines. 8

Unit III

4. (a) Find out the speed of sound in a gas. 8

(b) Show that for subsonic flow, the spherical disturbances speed throughout the entire field, whereas for supersonic flow, the disturbances are confined to the interior of the cone, the region outside the cone being unaffected by the disturbances. 8

5. (a) Discuss the flow through a nozzle. 8

(b) What is Isentropic Gas Flow ? Discuss. 8

Unit IV

6. (a) Find out the rates of strain for two-dimensional motion. 8

(b) Find out the Navier-Stokes equations of motion in cylindrical coordinates. 8

7. (a) Define Couette's flow and discuss the motion for it. 8

(b) Discuss the steady flow in tubes of uniform cross-section. 8

Unit V

8. (a) Define inspection analysis in the case of incompressible viscous fluid flow. 8

(b) *Prove that the discharge over a spillway is given by the relation :*

$$Q = VD^2 f\left(\frac{\sqrt{gD}}{V}, \frac{H}{D}\right).$$

where V = velocity of flow, D = depth at throat, H = heat of water, g = acceleration due to gravity. **8**

9. (a) Find out the Blasius solution for the boundary layer along a flat plate. **8**

(b) Define Displacement thickness and momentum thickness and show that : **8**

(i) $\int_0^{\delta} \frac{u}{U} dy = \delta - \delta_1$

(ii) $\int_0^{\delta} \left(\frac{u}{U}\right)^2 dy = \delta - \delta_1 - \delta_2$.