S.C.No.—21703401

M. Sc. EXAMINATION, 2023

(Fourth Semester)

(2021) (Main/Re-appear/Improvement)

MATHEMATICS

21MTH-401

Functional Analysis

Time : 3 Hours *Maximum Marks* : 80

Note : Attempt any *Five* questions. All questions carry equal marks.

Unit I

1. (a) Prove that Cauchy sequence in normed spaces is bounded.

(b) Show that norm is a continuous function.

(c) Write down the dual spaces of L^1 and L^p with details.

(d) Define unitary and normal operators.

(e) State Uniform bounded principal.

(f) Give an example of complete normed linear space which is not reflexive.

(g) Write down the properties of resolvent and spectrum.

(h) Define positive operators. 2×8=16

Unit II

2. (a) Prove that sequence Co is complete. 8

(b) Show that the normed space X is a Banach space if and only if every absolutely convergent series in X is convergent in X. 8

3. (a) A Banach space with a basis is separable. 8

(b) State and prove Equivalent Norms theorem. 8

Unit III

4. (a) A Banach space is a Hilbert space iff parallelogram law holds. 8

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(b) A subspace Y of a Hilbert space H is closed in H iff $Y = Y^{\perp\perp}$. 8

5. (a) If A is an positive operator on H then I + A is non-singular. In particular I + TT* and I + T*T are non- singular for any operator T on H. 8

(b) The Hilbert-adjoint operator T* of T in the Hilbert adjoint operator exist is unique and is a bounded linear operator with norm $\|T^*\| = \|T\|$. 8

Unit IV

6. (a) Prove that every vector space $X \neq \{0\}$ has a Hamel Basis. 8

(b) Let X be a normed space and let $x_0 \neq 0$ be any element of X then there exaists a bounded linear functional f on X such that $\|\tilde{f}\| = 1, \tilde{f}(x_0) = \|x_0\|$. 8

7. (a) Let B and B' be two Banach spaces and T is a linear transformation from B to B' then T is continuous iff its graph is closed. 8

(b) State and prove open mapping theorem. 8

Unit V

8. State and prove Spectral theorem for finite dimensional normed spaces. 16

9. (a) If T is a bounded linear operator on a complex Banach space then for the spectral radius $r_\sigma(T)$ of T we have

$$r_\sigma(T) = \lim_{x \to \infty} \sqrt[n]{\|T\|^n}.$$ 8

(b) Let $T \in B(H_1, H_2)$, then

(i) T is compact ↔ T*T or TT* is compact

(ii) T is compact ↔ T*is compact. 8