Roll No. _____

S.C.No.—21703402

**M. Sc. EXAMINATION, 2023**

(Fourth Semester)

(2021) (Main/Re-appear/Improvement)

MATHEMATICS

21MTH-402

Integral Equations

*Time :* 3 *Hours* *Maximum Marks :* 80

**Note :** Attempt any *Five* questions. All questions carry equal marks.

1. (a) Define Homogeneous integral equations, singular integral equation and Integro-differential equation.

(b) State Cauchy's Formula for Repeated Integration.

(c) Define Initial value and boundary value problems,





(d) What do you mean by Fredholm Alternative ? State the theorem of Fredholm Alternative.

(e) Define Degenerate Kernel and Symmetric Kernel.

(f) State basic properties of Green's function.

(g) State Abel's Integral equation and write the appropriate method to solve the same.

(h) State Convolution theorem for Laplace transformation and Fourier transformation.

**Unit I**

2. (a) Find the resolvent kernel of the Volterra integral equation with the kernel :

$$K(x,t)=e^{x-t}$$

(b) Find the Neumann series for the solution of the integral equation :

$$y(x)=1+\int_0^x xt\, y(t)\, dt$$

3. (a) Solve :

$$y(x) = \cos x - x - 2 + \int_0^x (t - x)\, y(t)\, dt$$

(b) Solve the following Volterra integral equation of the first kind by first reducing it to Volterra integral equation of second kind :

$$\int_0^x \left(1 - x^2 + t^2\right) y(t)\, dt = \frac{x^3}{2}$$

**Unit II**

4. (a) Solve the Fredholm integral equation :

$$y(x) = x + \lambda \int_0^1 \left(xt^2 + x^2 t\right) y(t)\, dt$$

(b) Solve :

$$y(x) = \lambda \int_0^{\pi} \left(\cos^2 x \cos 2t + \cos 3x \cos^3 t\right) y(t)\, dt$$

5. (a) Solve the following integral equation by method of iterated kernels :

$$y(x) = 2 + \lambda \int_0^1 \sin(x - 2t)\, y(t)\, dt$$




(b) Solve the following integral equation by Method of Fredholm Resolvent Kernel as a ratio of two series :

$$y(x) = f(x) + \lambda \int_0^1 xe^t y(t)\, dt$$

**Unit III**

6. (a) Find the Green's function for $y''(x) - y(x); y(0) = y(1) = 0$ using Variation of parameters.

(b) Reduce the boundary value problem $\frac{d^2y}{dx^2} + \lambda y = x, 0 < x < \frac{\pi}{2}; y(0) = y\left(\frac{\pi}{2}\right) = 0$ into Fredholm integral equation with Kernel as Green's function.

7. (a) Solve the Fredholm integral equation of second kind :

$$y(x) = x + \lambda \int_0^1 K(x, t)\, y(t)\, dt$$

with symmetric kernel :

$$K(x, t) = \begin{cases} x(1-t), & 0 \le x \le t \\ t(1-x), & t \le x \le 1 \end{cases}$$

(b) Show that the eigen functions of symmetric kernel corresponding to two distinct eigen values are orthogonal.

## Unit IV

8. (a) Solve the following singular integral equation $f(x)=\int_0^x \frac{u(t)}{(x-t)^{\alpha}}dt, 0<\alpha<1$ using Laplace transformation.

(b) Solve the following integro-differential equation : 

$$u'(x)+5\int_0^x \cos 2(x-t)u(t)\,dt=10$$

where $u(0)=2$.

9. (a) Find the resolvent kernel of the following Volterra Integral equation and hence find their solution :

$$u(x)=f(x)+\int_0^x (x-t)u(t)\,dt$$





(b) Solve the following integral equation for $f(x)$ :

$$\int_0^{\infty} f(x)\sin sx\,dx=\begin{cases}1, & 0\le s\le 1\\ 2, & 1\le s<2\\ 0, & s>2\end{cases}$$